Appendix A Formal Proofs

Lemma 1. *Given a CTS $M_P = (\Sigma, S, s_0, L, \delta, P, L_P)$ with parameters $A, B \in P$, if $B \rightsquigarrow_c A$ for some parameter configuration c, then there does not exist any possible run of M_P with prefix $\alpha = s_0 \xrightarrow{*} s_i \xrightarrow{\nu_B} s_j \xrightarrow{*} s_k \xrightarrow{\nu_A} s_l$, where ν_A and ν_B are guards over A and B, resp., and $s_i, s_j, s_k, s_l \in S$, i.e., a transition with guard over B appears before a transition with guard over A.*

Proof. Follows from Definition 6. Let Π_P be the set of all possible runs of CTS M_P. Let $\pi_p \in \Pi_P$ be a possible run with prefix $s_0 \xrightarrow{*} s_i \xrightarrow{\nu_B} s_j \xrightarrow{*} s_k \xrightarrow{\nu_A} s_l$. In configured run $\pi_{P(c)}$, transition with guard over B is reachable irrespective of the setting for A, violating our premise of $B \rightsquigarrow_c A$. Therefore, when $B \rightsquigarrow_c A$, a transition with guard over B never appears before a transition with guard over A in all possible runs. $\square$

Theorem 2 (Redundant Instances). *Given a CTS $M_P = (\Sigma, S, s_0, L, \delta, P, L_P)$ with parameters $A, B \in P$ such that $B \rightsquigarrow_c A$ for some configuration c, and a LTL property φ, there exist configurations $c_1, c_2, \ldots c_k \in \mathbb{C}$ for $k = |B|$ such that*

- *$c_i(A) = c(A)$ for $0 < i \leq k$, and*
- *$c_i(B) = d_{B_i} \in [\![B]\!]$ for $0 < i \leq k$ and $[\![B]\!] = \{d_{B_1}, d_{B_2}, \ldots, d_{B_k}\}$*

For such configurations $M_{P(c_1)} \models \varphi \equiv M_{P(c_2)} \models \varphi \equiv \ldots \equiv M_{P(c_k)} \models \varphi$.

Proof. From Lemma 1 we know that if $B \rightsquigarrow_c A$, then a transition with guard over B never appears before a transition with guard over A in all possible runs of M_P. Also, from Definition 6 we know that if $B \rightsquigarrow_c A$ then there exists a parameter setting for A that makes transitions with guard over B unreachable in configured runs of M_P. Let this setting be $c(A) = d_A$ for $d_A \in [\![A]\!]$. For parameter configurations $c_1, c_2, \ldots c_k \in \mathbb{C}$ such that $c_i(A) = c(A)$ for $0 < i \leq k$, execution never reaches the transition with guard over B in all c-runs $\pi_{P(c_i)}$ (if it did, $B \not\rightsquigarrow_c A$). Irrespective of the setting to B, the same set of states are reachable, and c-runs $\pi_{P(c_i)}$, for $0 < i \leq k$, are identical. Therefore, $M_{P(c_1)} \models \varphi \equiv M_{P(c_2)} \models \varphi \equiv \ldots \equiv M_{P(c_k)} \models \varphi$. $\square$

Lemma 2. FindUP *returns unset parameters $P_i \in P$ for all reachable transitions $t \in \delta$ such that guard $L_P(t)$ is a guard over P_i, and is undefined.*

Proof. Consider a possible run $\pi_P = s_0 \xrightarrow{\nu_1} s_1 \xrightarrow{\nu_2} s_2 \ldots \xrightarrow{\nu_n} s_n$ such that $\hat{c} \vdash \nu_1$ and ν_2 is undefined. From Definition 4, transition $t = (s_1, s_2)$ is reachable, while all transitions after s_2 are not reachable. Hence, the unset parameter for guard ν_2 is added to the return set of FindUP. Depth-first traversal (DFT) allows scanning all possible runs of a CTS without enumerating all of them. The traversal backtracks whenever a transition with an undefined guard is visited. Therefore unset parameters for all edges $t \in \delta$ that can be reached starting from an initial node in DFT, and for which $L_P(t)$ is undefined, are returned by FindUP. $\square$

Theorem 3 (GenPC is sound). *Given a CTS M_P with parameters $A, B \in P$, if there exists a partial configuration $\hat{c} \in \hat{\mathbb{C}}$ with $\hat{c}(A) = d_{A_n} \in [\![A]\!]$ and B unset, then there exist configurations $c_1, c_2, \ldots c_k \in \mathbb{C}$ for $k = |B|$ such that*

- *$c_i(A) = \hat{c}(A)$ for $0 < i \leq k$, and*
- *$c_i(B) = d_{B_i} \in [\![B]\!]$ for $0 < i \leq k$ and $[\![B]\!] = \{d_{B_1}, d_{B_2}, \ldots, d_{B_k}\}$*

for which $B \rightsquigarrow_{c_i} A$.

Proof. We prove the contrapositive of the statement. When parameters A and B are not dependent, there is no possible run of M_P that contains transitions with guards over both A and B (follows from Lemma 1). Therefore, every possible run of M_P is of the form $\pi_P = s_0 \xrightarrow{\nu_1} s_1 \xrightarrow{\nu_2} \ldots \xrightarrow{\nu_n} s_n$ where all ν_i, for $0 < i \leq n$, are either *true* or guards over $P \setminus \{A, B\}$, or guards over either parameter A or B. A call to FindUP with a setting for A, returns unset parameter B (follows from Lemma 2) that is then set to every value in $[\![B]\!]$ domain in GenPC. Therefore, if A is set to $d_{A_n} \in [\![A]\!]$ in the call to FindUP, then $\hat{\mathbb{C}}$ contains $k = |B|$ partial configurations $\hat{c}_i$ such that

- $\hat{c}_i = d_{A_n}$ for $0 < i \leq k$, and
- $\hat{c}_i(B) = d_{B_i} \in [\![B]\!]$ for $0 < i \leq k$ and $[\![B]\!] = \{d_{B_1}, d_{B_2}, \ldots, d_{B_k}\}$

Therefore, when A and B are not dependent, for every setting of A, $\hat{\mathbb{C}}$ contains $|B|$ partial parameter configurations; one for every different setting of B. Hence, GenPC is sound. □

Theorem 4 (GenPC is complete). *Given a CTS M_P with parameters $A, B \in P$, if there exist configurations $c_1, c_2, \ldots c_k \in \mathbb{C}$ for $k = |B|$ such that*

- *$c_i(A) = d_{A_n}$ for $0 < i \leq k$ and $d_{A_n} \in [\![A]\!]$, and*
- *$c_i(B) = d_{B_i} \in [\![B]\!]$ for $0 < i \leq k$ and $[\![B]\!] = \{d_{B_1}, d_{B_2}, \ldots, d_{B_k}\}$*

for which $B \rightsquigarrow_{c_i} A$, then there exists a partial configuration $\hat{c} \in \hat{\mathbb{C}}$ with $\hat{c}(A) = d_{A_n}$ and B unset.

Proof. Let $A, B \in P$ be dependent parameters such that $B \rightsquigarrow_c A$ for some configuration c and $c(A) = d_{A_n} \in [\![A]\!]$. When $B \rightsquigarrow_c A$, there is no possible run of M_P in which a transition with guard over B appears before a transition with guards over A (follows from Lemma 1). A call to FindUP with a partial configuration $\hat{c}$ such that $\hat{c}(A) = d_{A_n}$ does not return B as an unset parameter (follows from Lemma 2). Therefore, GenPC generates a partial configuration $\hat{c} \in \hat{\mathbb{C}}$ with $\hat{c}(A) = d_{A_n}$ and B unset. Hence, GenPC is complete. □

Theorem 5 (Minimality). *The minimal set of parameter configurations is $\hat{\mathbb{C}}$.*

Proof. Suppose towards contradiction that $\hat{\mathbb{C}}$ is not minimal. Then there is a minimal set of configurations $\hat{\mathbb{C}}^*$ with $\hat{\mathbb{C}}^* \subset \hat{\mathbb{C}}$. Take $c \in \hat{\mathbb{C}} \setminus \hat{\mathbb{C}}^*$. Now $c \notin \hat{\mathbb{C}}^*$ implies that there exists a $c_i \in \hat{\mathbb{C}} \cap \hat{\mathbb{C}}^*$ for which $B \rightsquigarrow_{c_i} A$ with $c_i(A) = c(A)$ and $c_i(B) \neq c(B)$, i.e., the setting of A in c_i makes transitions with guards over B unreachable and hence the setting of B does not effect configured runs. Since $\hat{\mathbb{C}}$ contains both c and c_i, then from the correctness of GenPC, $B \not\rightsquigarrow_{c_i} A$ (follows from Theorem 3 and Theorem 4). This is a contradiction, thus $\hat{\mathbb{C}}$ must be minimal. □

Theorem 6 (Property Dependence). *For two LTL properties φ_1 and φ_2 dependence can be established by model checking with universal model U.*

Proof. There are a total of four cases to consider. We only show proof for $(\varphi_1 \rightarrow \varphi_2)$, since other dependencies follow a similar proof. From Theorem 1 we know that a LTL formula φ is satisfiable iff $U \not\models \neg\varphi$. Therefore, φ is unsatisfiable iff $U \models \neg\varphi$. Let $\varphi = \neg(\varphi_1 \rightarrow \varphi_2)$. Therefore, if $U \models (\varphi_1 \rightarrow \varphi_2)$ then $\neg(\varphi_1 \rightarrow \varphi_2)$ is unsatisfiable or $(\varphi_1 \rightarrow \varphi_2)$ is valid, and vice-versa. $\square$