Satisfiability Checking
A Formal Method Unto Itself

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Applied Formal Methods

Satisfiability Checking

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Satisfiability checking can be used to solve problems all by itself!

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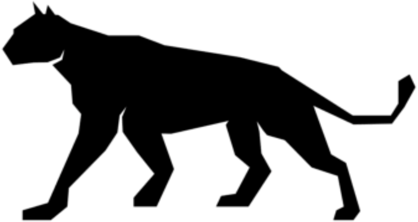
Formula re-writing in this way is great for *specification debugging* and for *problem solving*!

The Lady and the Tiger

- Two doors containing either Ladies or Tigers



The Lady and the Tiger

- You will be shown two doors, to two rooms.
  - Each could contain either a lady or a tiger . . .
  - It could be that **both** rooms contain a lady, or that **both** rooms contain a tiger!
- You will need to reason carefully and logically to survive!
- For each question, pick a door, or decide not to open a door.
  - +1 point for picking a lady or refusing to pick if both doors contain tigers.
  - -1 point for getting eaten by a tiger

The Lady and the Tiger
Q1

One of these is true…

In this room, there is a lady, and in the other room there is a tiger.

The other is false…

In one of these rooms there is a lady, and in one of these rooms there is a tiger.

Proof by Truth Table

Let's call the room on the left Room 1 and the room on the right Room 2
L1/L2 is True when there is a Lady in Room 1/2 and False otherwise
S1 is True when Sign 1 is True and False otherwise (when Sign 2 is True)

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T	T	F	F	F	F	F
T	F	T	T	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	**T**	F	F	T	F	**T**
F	F	T	F	F	F	F
F	F	F	F	F	F	F

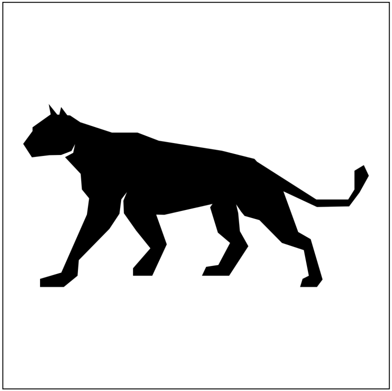
Many Proof Methods Work!

There is an easier proof. . . proof by contradiction (aka RAA)!

Assume there is a lady in Room 1. . .

The Lady and the Tiger
Q1

One of these is true...



The other is false...

The Lady and the Tiger
Q2

Either both signs are false…

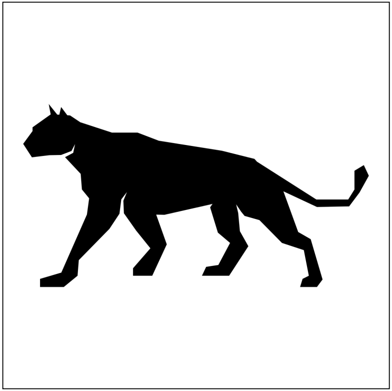
At least one of these rooms contains a lady

Or both are true…

A tiger is in the other room…

The Lady and the Tiger
Q2

Either both signs are false...



Or both are true...

The Lady and the Tiger
Q3

Either both signs are false…

Either a tiger is in this room, or a lady is in the other room.

Or both are true…

An lady is in the other room.

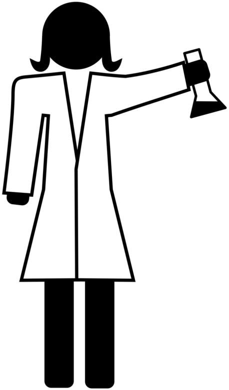
The Lady and the Tiger
Q3

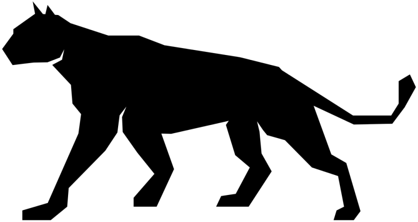
Either both signs are false… Or both are true…

Let's try a hard lady and tiger puzzle.

The Lady and the Tiger

- Doors containing either Ladies or Tigers





The Lady and the Tiger

- Once again, you'll have to tell Ladies from Tigers
- A new twist is being added.
  - Two doors
  - If a lady is in Room I, then the sign on the door is true. If a tiger is in Room I, then the sign on the door is false.
  - The opposite is true for Room II.

  - **Problem… the signs fell off the doors. We can't remember which one goes where!!!**

The Lady and the Tiger

Q1

Room ??

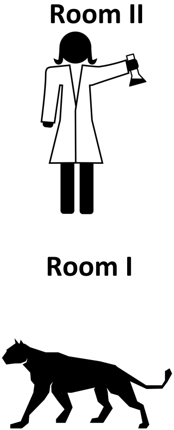
This room contains a tiger.

Room ??

Both rooms contain tigers

The Lady and the Tiger

Q1



The Lady and the Tiger

- Once again, you'll have to tell Ladies from Tigers
- A new twist is being added.
  - **Three** doors!
  - One lady, **TWO** tigers
  - At most one of the signs is true

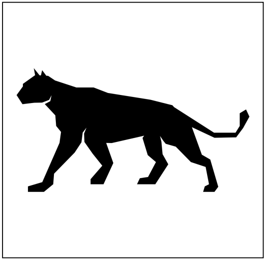
The Lady and the Tiger
Q2

Room I	Room II	Room III
A tiger is in this room.	A lady is in this room.	A tiger is in room II.

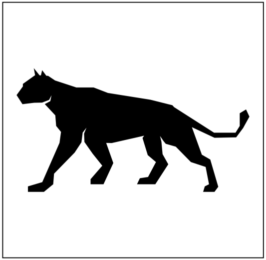
The Lady and the Tiger
Q2

Room I

Room II



Room III



The Lady and the Tiger

- Once again, you'll have to tell Ladies from Tigers
- A new twist is being added.
  - Three doors!
  - One lady, TWO tigers
  - The sign on the door of the room with the lady is true. At least one of the other two signs is false!

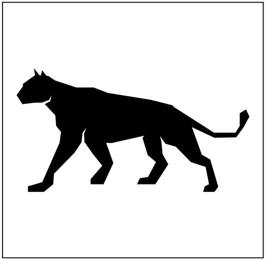
The Lady and the Tiger
Q3

Room I	Room II	Room III
A tiger is in room II.	A tiger is in this room.	A tiger is in room I.

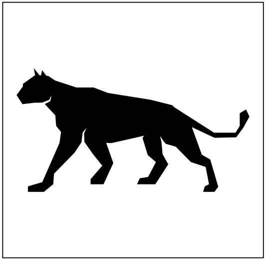
The Lady and the Tiger
Q3

Room I

Room II



Room III



The Lady and the Tiger

- Only one room contains a lady. Every other room has a tiger, or is empty.
- The sign of the door with the lady is true. The signs of all rooms with tigers are false. Empty room's signs can be true or false.
- This problem would be unsolvable, however, if you knew if Room VIII was empty or not, you'd be able to figure out which room had the lady.

  And knowing that this information would make the puzzle solvable, means you can solve the puzzle without even knowing the answer to that question.

Room I

The lady is in an odd numbered room.

Room II

This room is empty.

Room III

Either Sign V is right or Sign VII is wrong.

Room IV

Sign I is wrong.

Room V

Either Sign II or Sign IV is right.

Room VI

Sign III is wrong.

Room VII

The lady is not in Room I

Room VIII

This room contains a tiger and room IX is empty.

Room IX

This room contains a tiger and VI is wrong.