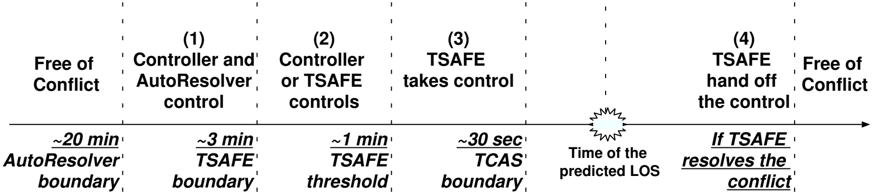
SAT Applications in Temporal Logics

Kristin Yvonne Rozier
Iowa State University

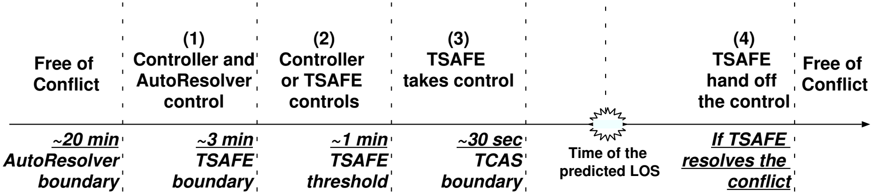
SAT/SMT/AR Summer School
June 26, 2024

AAC Operational Concept[1]



[1] H Erzberger, K Heere. "Algorithm and operational concept for resolving short-range conflicts." Proc. IMechE G J. Aerosp. Eng. 224 (2) (2010) 225–243.

AAC Operational Concept[2]



Formal verification triggered system design changes[1]

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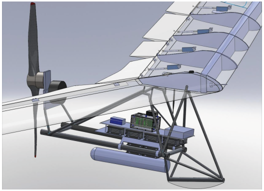
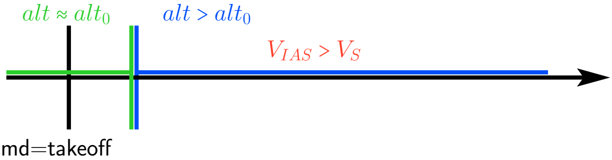
[1] Y. Zhao and K.Y. Rozier. "Formal Specification and Verification of a Coordination Protocol for an Automated Air Traffic Control System." SCP Journal, vol-96, no-3, pg 337-353, 2014.

[2] H Erzberger, K Heere. "Algorithm and operational concept for resolving short-range conflicts." Proc. IMechE G J. Aerosp. Eng. 224 (2) (2010) 225–243.

Operational Concept for the Swift UAS





Whenever the Swift UAS is in the air, its indicated airspeed (V_{IAS}) must be greater than its stall speed V_S. The UAS is considered to be air-bound when its altitude *alt* is larger than that of the runway alt_0.[3]

[3] T. Reinbacher, K.Y. Rozier, J. Schumann. "Temporal-Logic Based Runtime Observer Pairs for System Health Management of Real-Time Systems." TACAS 2014.

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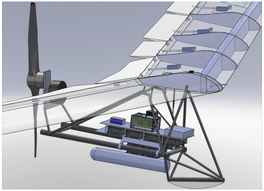

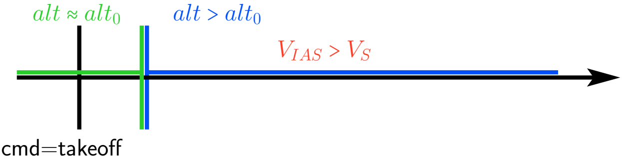

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$$\textsc{Always}((alt > alt_0) \rightarrow (V_{IAS} > V_S))$$

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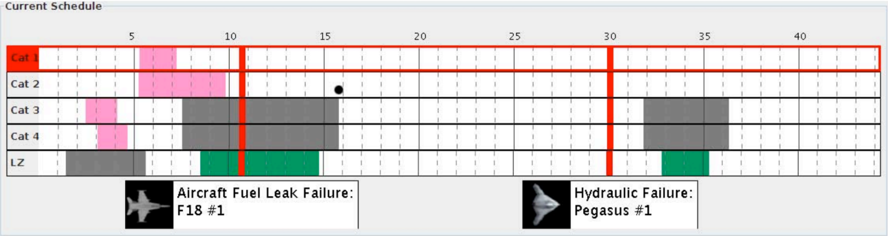
There is a Pattern Here. . .



Air Force aircraft carrier deck scheduling: deck resource timeline displaying three failures[4]

[4] J.C.Ryan, M.L.Cummings, N.Roy, A Banerjee, A.Schulte. "Designing an Interactive Local and Global Decision Support System for Aircraft Carrier Deck Scheduling." AIAA Infotech, 2011.

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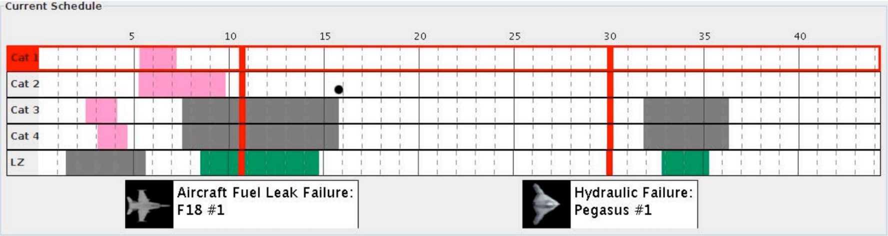

Air Force aircraft carrier deck scheduling: deck resource timeline displaying three failures[4]

Aerospace Operational Concepts Are Often Specified With Timelines

[4] J.C.Ryan, M.L.Cummings, N.Roy, A Banerjee, A.Schulte. "Designing an Interactive Local and Global Decision Support System for Aircraft Carrier Deck Scheduling." AIAA Infotech, 2011.

A Natural Logic for Operational Timelines: Linear Temporal Logic

Linear Temporal Logic (LTL) formulas reason about linear timelines:

- finite set of atomic propositions {p q}
- Boolean connectives: $\neg$, $\wedge$, $\vee$, and $\rightarrow$
- temporal connectives:

$\mathcal{X}p$	NEXT TIME
$\Box p$	ALWAYS
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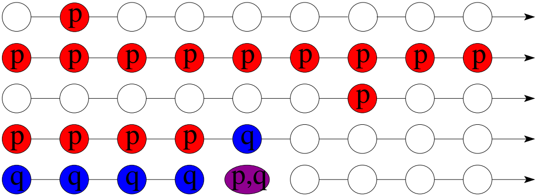
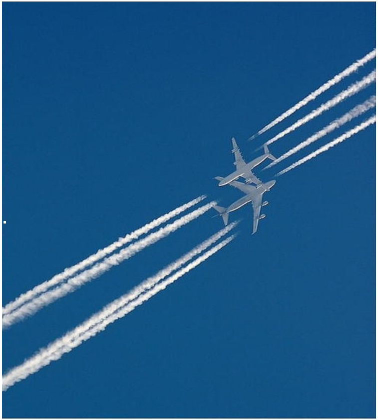


Formal Verification Via Model Checking

1. Describe system requirements in a formal specification, φ.
2. Create a system model with formal semantics, M.
3. Check that M satisfies φ.



Model checking finds disagreements between the system model and the formal specification.

Formal Verification Via LTL Model Checking

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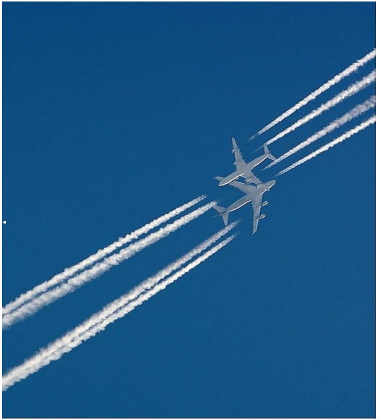

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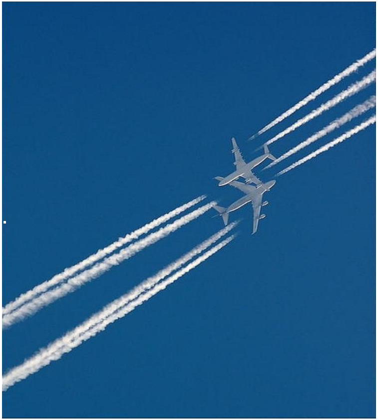
   - Graph-search-based
   - BDD-based
   - BMC-based
   - IC3-based



Model checking finds disagreements between the system model and the formal specification.

Formal Verification Via LTL Model Checking

1. Describe system requirements in a formal LTL specification, φ.
   - Only works if the formula is correct!
2. Create a system model with formal semantics, M.
3. Check that M satisfies φ.
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   - BDD-based
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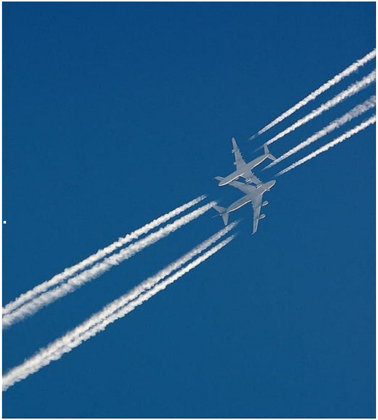

Model checking finds disagreements between the system model and the formal specification.

Property Assurance: We Propose Satisfiability Checking

$M \models \varphi$ may not mean the system has the intended behavior

Recall that a property φ is *valid* iff $\neg\varphi$ is *unsatisfiable*.

If $\neg\varphi$ is not satisfiable, then

- There can never be a counterexample.
- Model checkers will always return "success."
- φ is probably wrong.

Property Assurance: We Propose Satisfiability Checking

$M \models \varphi$ may not mean the system has the intended behavior

$M \not\models \varphi$ may not mean the system does not have the intended behavior

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If φ is not satisfiable, then

- There is always a counterexample.
- Model checkers will always return "failure."
- φ is probably wrong.

Specification Debugging: LTL Satisfiability Checking

For each property φ and $\neg\varphi$ we should check for satisfiability.

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LTL-to-Automaton Complexity

- LTL property f of size $|\varphi|$
- System model M of size $|M|$
- LTL satisfiability checking takes time $|M| \cdot 2^{\mathcal{O}(|\varphi|)}$.

LTL Satisfiability Checking is PSPACE-Complete!

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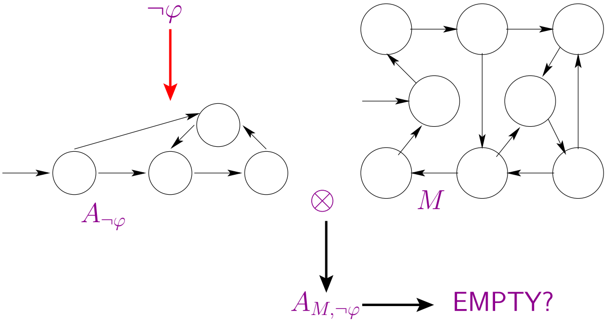
LTL Satisfiability Checking is PSPACE-Complete!

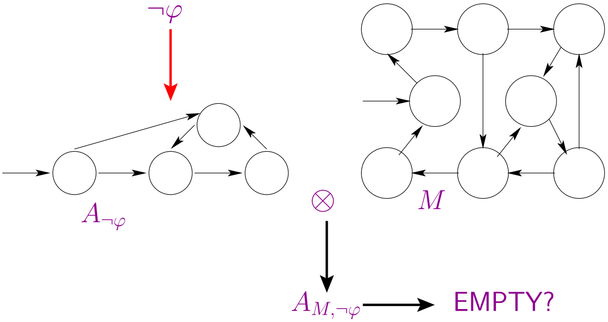
We have to be smart about encoding the problem!

Ex: Automata-Theoretic Approach to Model Checking: One of the PSPACE-Complete Algorithms for LTL-SAT

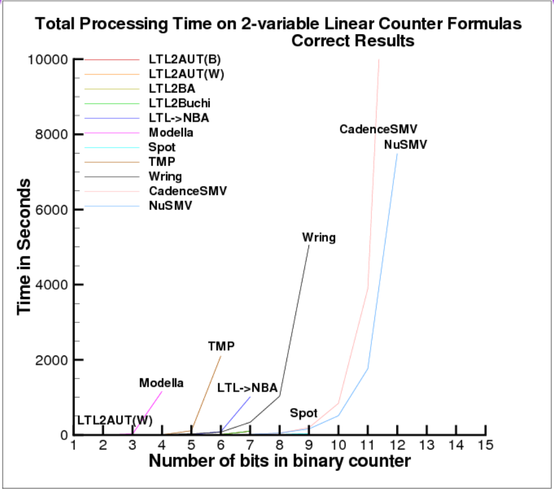


Ex: Automata-Theoretic Approach to Model Checking: One of the PSPACE-Complete Algorithms for LTL-SAT

Requires efficient LTL-to-automaton translation.



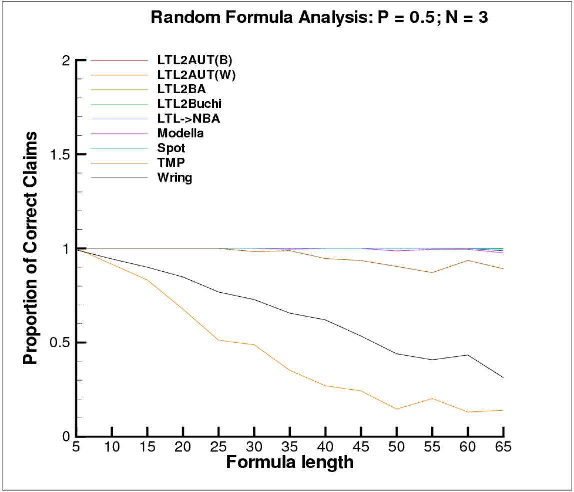
LTL Satisfiability is Hard to Scale[5]



Many tools cannot check 8-bit binary counter formulas

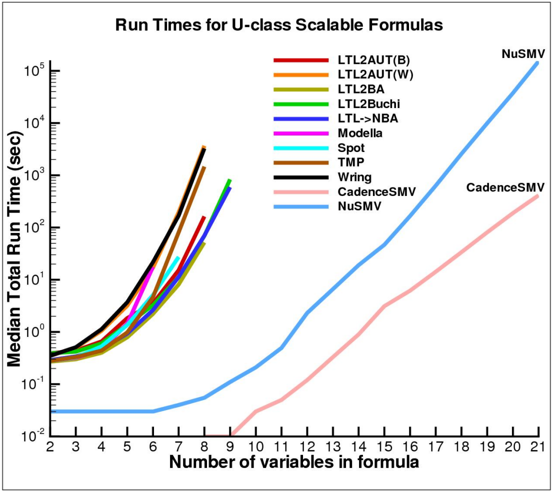
[5] K.Y.Rozier, M.Y.Vardi. "LTL Satisfiability Checking." STTT Journal, pg. 123–137, 2010.

LTL Satisfiability is Hard to Code Correctly[6]



6 K.Y.Rozier, M.Y.Vardi. "LTL Satisfiability Checking." STTT Journal, pg. 123–137, 2010.

Implementation is Hugely Influential[7]



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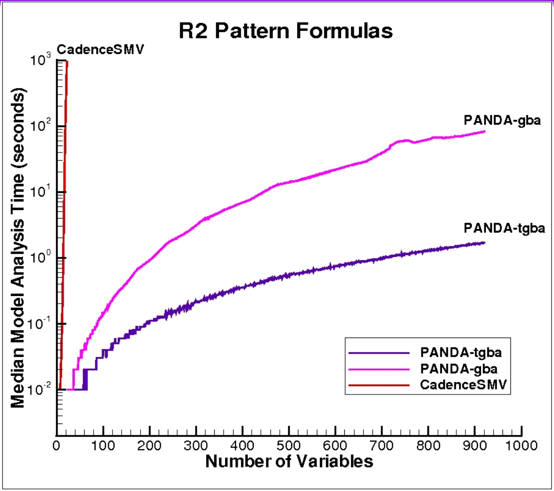
Better Encoding Can Lead to Exponential Improvement! [8]



$$R_2(n) = (..(p_1 \mathcal{R}\ p_2)\ \mathcal{R}\ \ldots)\ \mathcal{R}\ p_n.$$

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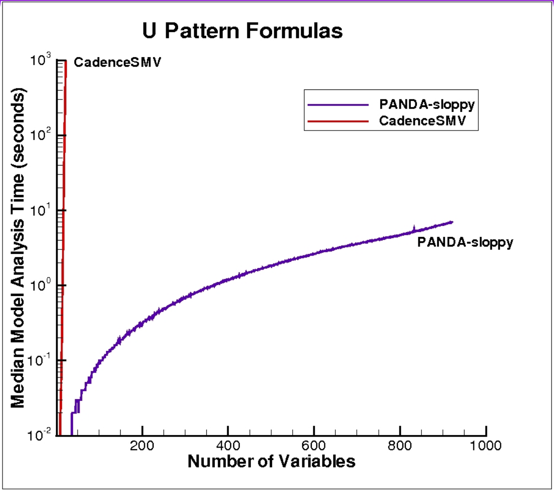
Even for Very Hard Formulas! [9]



$$U(n) = (\dots(p_1 \,\mathcal{U}\, p_2)\,\mathcal{U}\, \dots)\,\mathcal{U}\, p_n.$$

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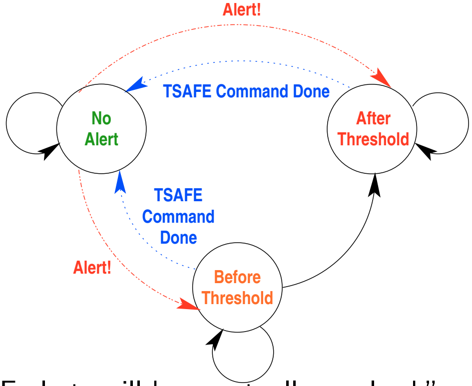
LTL Satisfiability Checking Found A Specification Bug

LTL safety requirement φ_0
LTL fairness constraint φ_1

ALWAYS EVENTUALLY $\varphi_1 \rightarrow \varphi_0$

An overstrict φ_1 can effectively cause φ_0 to be valid!



Example:

Safety Requirement: "All TSAFE alerts will be eventually resolved."
Fairness Constraint: Progress between TSAFE alerts

Wrong: `FAIRNESS (TSAFE_Alert = Non);`
Right: `FAIRNESS (TSAFE_Alert != AT);`

LTL-SAT Problem Examples

- **Specification Debugging:** If the conjunction of all properties is not satisfiable, where is the problem?

[11] G. Hariharan, P. H. Jones, K. Y. Rozier, T. Wongpiromsarn. "Maximum Satisfiability in Mission-time Linear Temporal Logic." FORMATS, 2023.

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LTL-SAT Problem Examples

- **Specification Debugging:** If the conjunction of all properties is not satisfiable, where is the problem?
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These are all MAX-SAT![11][12]

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Linear Temporal Logic: Reasons Over Infinite Traces

Linear Temporal Logic (LTL) formulas reason about linear timelines:

- finite set of atomic propositions {p q}
- Boolean connectives: ¬, ∧, ∨, and →
- temporal connectives:

$\mathcal{X}p$	NEXT TIME
$\Box p$	ALWAYS
$\Diamond p$	EVENTUALLY
$p\mathcal{U}q$	UNTIL
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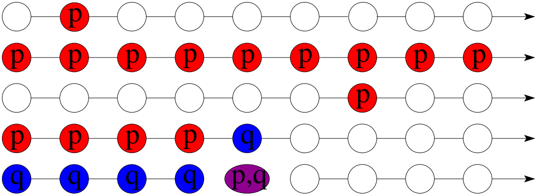
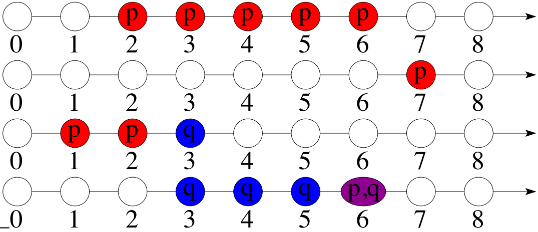


MLTL: A Good Specification Language[13]

Mission-Time Temporal Logic (MLTL) reasons about *integer-bounded* timelines:

- finite set of atomic propositions {p q}
- Boolean connectives: $\neg$, $\wedge$, $\vee$, and $\rightarrow$
- temporal connectives *with time bounds*:

Symbol	Operator	Timeline
$\Box_{[2,6]} p$	ALWAYS$_{[2,6]}$	
$\Diamond_{[0,7]} p$	EVENTUALLY$_{[0,7]}$	
$p \mathcal{U}_{[1,5]} q$	UNTIL$_{[1,5]}$	
$p \mathcal{R}_{[3,8]} q$	RELEASE$_{[3,8]}$	



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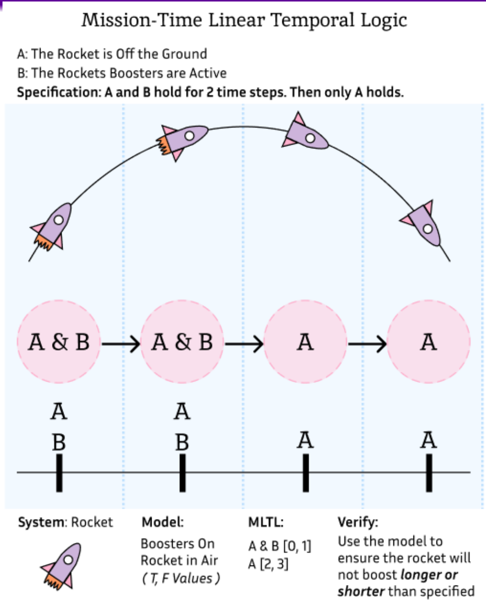
Mission-Time Linear Temporal Logic
A: The Rocket is Off the Ground
B: The Rockets Boosters are Active
Specification: A and B hold for 2 time steps. Then only A holds.
A & B
A & B
A
A
A
B
A
B
A
A
System: Rocket
Model:
Boosters On
Rocket in Air
(T, F Values)
MLTL:
A & B [0, 1]
A [2, 3]
Verify:
Use the model to ensure the rocket will not boost longer or shorter than specified

MLTL Runtime Benchmark Generation: An Easier Problem[14]

Time:

0	1	2	3	4	5	6	7	8	9

MLTL formula φ evaluated over system trace π:

$$\forall i : 0 \leq i \leq \text{MissionTime } \pi, i \models \varphi.$$

An MLTL Runtime Benchmark is a 3-tuple:

- Input stream, or computation, π
- MLTL formula, φ, over n propositional variables
- Oracle $\mathcal{O}$, of $\langle time, verdict \rangle$

[14] J.Wallin and K.Y.Rozier. "Generating System-Agnostic Runtime Verification Benchmarks from MLTL Formulas via SAT." Under Submission, 2018.

MLTL Runtime Benchmark Generation: An Example[15]

Time:	a	$\neg a$	$\neg a$	a	a	a	a	a	a	a
	0	1	2	3	4	5	6	7	8	9

MLTL formula φ evaluated over system trace π:

$$\forall i : 0 \leq i \leq \text{MissionTime } \pi, i \models \varphi.$$

MLTL Runtime Benchmark Example:

- $\pi = a, \neg a, \neg a, a, a, a, a, a, a, a$
- $\varphi = \textsc{Always}_{[5]}(a)$
- $\mathcal{O} = \langle 0, F \rangle, \langle 1, F \rangle, \langle 2, F \rangle, \langle 3, T \rangle, \langle 4, T \rangle, \ldots$

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	0	1	2	3	4	5	6	7	8	9

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A SAT Encoding:

Assign a_i to a at time i.

Iteratively conjunct the satisfying assignment from i to the formula for $i + 1$. Record UNSAT as $\mathcal{O} = \langle i, F \rangle$; otherwise $\langle i, T \rangle$

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So where do we use this IRL?

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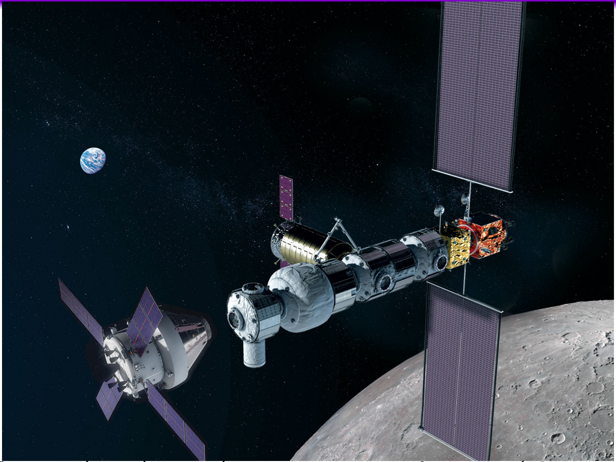
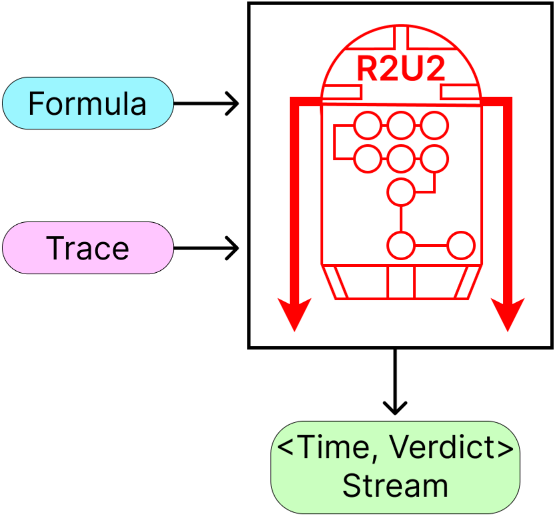
NASA Lunar Gateway: Assume-Guarantee Contracts[16]



$$(CMD == START) \rightarrow (\Box_{[0,5]}(ActionHappens\& \Box_{[0,2]} (CMD = END)))$$

[16] Dabney, James B., Julia M. Badger, and Pavan Rajagopal. "Adding a Verification View for an Autonomous Real-Time System Architecture." In AIAA Scitech 2021 Forum, p. 0566. 2021.



[17] [18]

[17] K. Y. Rozier, J. Schumann. "R2U2: Tool Overview." RV-CUBES, 2017.

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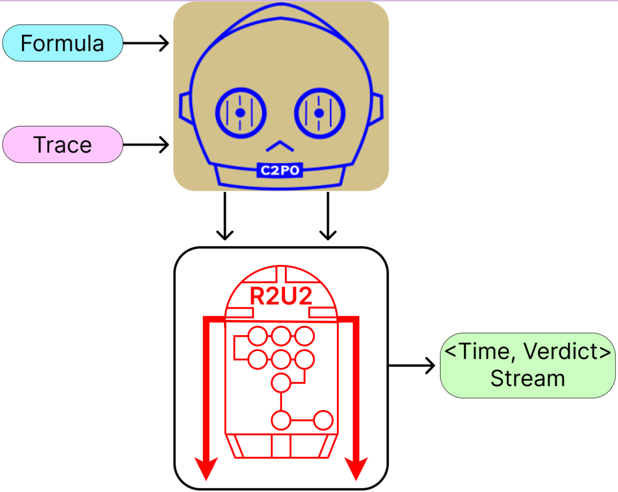

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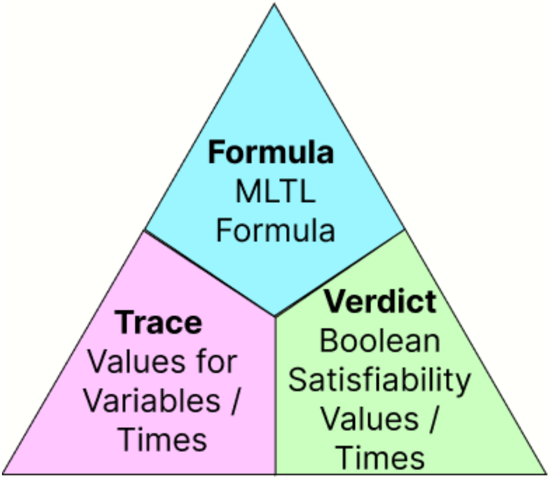
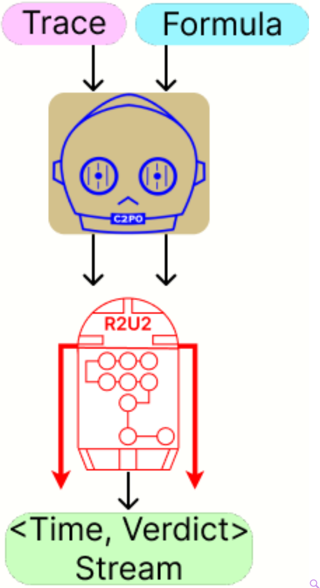
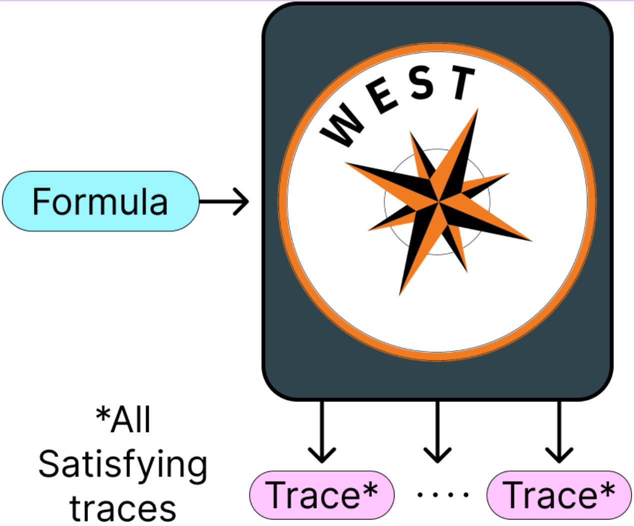


Given any 2 you can find the 3rd





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[21] J. Elwing, L. Gamboa-Guzman, J. Sorkin, C. Travesset, Z. Wang, K. Y. Rozier. "Mission-Time LTL (MLTL) Formula Validation via Regular Expressions." In *integrated Formal Methods* (iFM), 2023.

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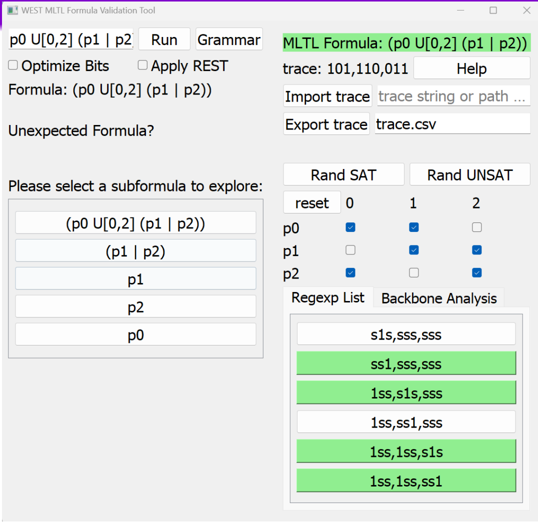

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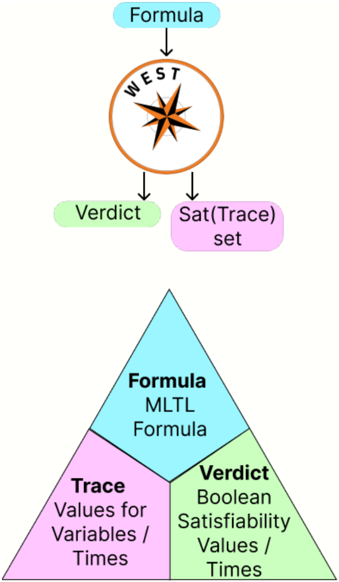
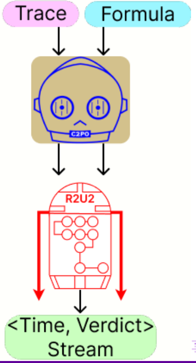
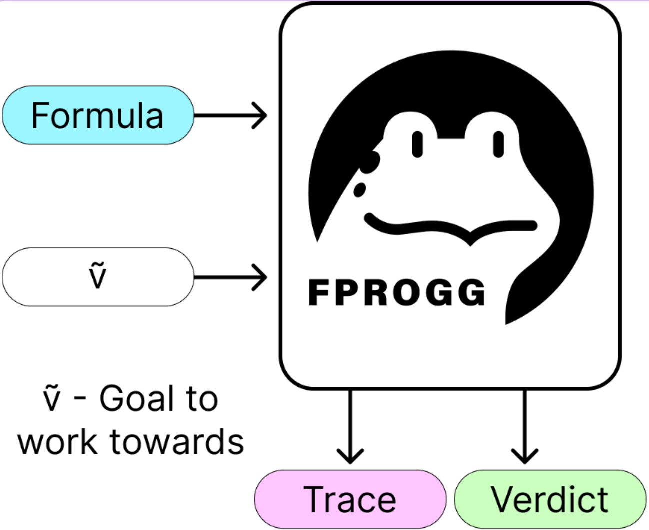




Given any 2 you can find the 3rd



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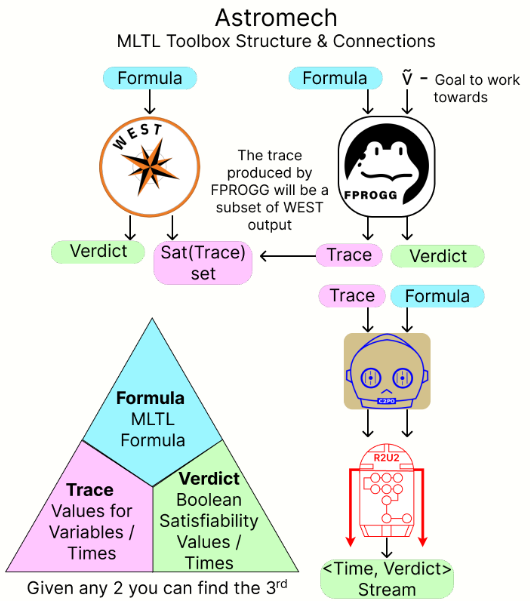
Astromech
MLTL Toolbox Structure & Connections
Formula
Formula
Ṽ - Goal to work towards
WEST
The trace produced by FPROGG will be a subset of WEST output
FPROGG
Verdict
Sat(Trace) set
Trace
Verdict
Trace
Formula
C2PO
R2U2
Formula MLTL Formula
Trace Values for Variables / Times
Verdict Boolean Satisfiability Values / Times
<Time, Verdict> Stream
Given any 2 you can find the 3rd